Linear dynamical system identification

... basic elements and Labs guidelines

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February 20, 2026

Introduction

Course plan, overview and questions addressed

L1: Overview, signals construction, pre-treatment and non-parametric analysis

- Realization, transfer functions, ...
- Construct an experimental plan and signals.
- How to analyze signals, and derive some properties?

L2: Data-driven model construction in the time- and frequency-domain

- Construct a linear model from time- or frequency-domain data.
- How much is it valid? How to validate, discuss, amend it?

L3: L2 cont'd & Labs guidelines

- Illustration in practice
- Experimental setup & numerical tools presentation.
- Methodology for the lab.

L3: L2 cont'd & Labs guidelines

- Experimental setup & numerical tools presentation.
- Methodology for the lab.

Notions treated

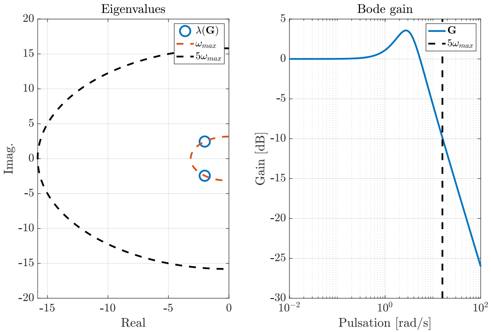
- A detailed illustration of L1-L2
- Presentation of the experimental test benches
- Talk and learn from user (setup, measurements, inputs, disturbances, knowledge, stability, etc.)
- Getting started and method (code, report, organisation, etc.)
- Group evaluation

Content

A detailed identification example #1

Setting (unknown setup)

```
G      = tf([1 -2],[.1 .4 1]); G = G/dcgain(G);
Ns     = 2^8;
fmax   = 5*max(abs(eig(G)))/2/pi;
Fs     = 2^(nextpow2(fmax)+1);
Ts     = 1/Fs;
```



Remarks

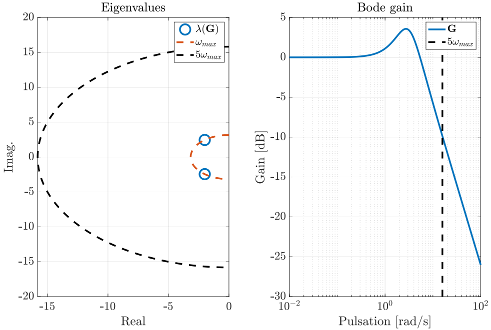
We do not know $\mathbf{G}$, but

- Analyze (visually) the system and ask user if possible
- Input/output range, type, unit...
- Evaluate time response
  $\rightarrow$ gives an idea of f_{max}
  $\rightarrow T_s = \dfrac{1}{f_s} \approx \dfrac{1}{5 f_{max}}$

A detailed identification example #1

Setting (unknown setup)

```
% Noise model generator
n          = 1e-1; % noise amplitude in percent
Gn         = tf(n,[1/100 1]);
```



Remarks

Noise generator for this numerical test

- gain $0.1 \approx 10\%$
- limited frequency range $\approx 50/\pi$

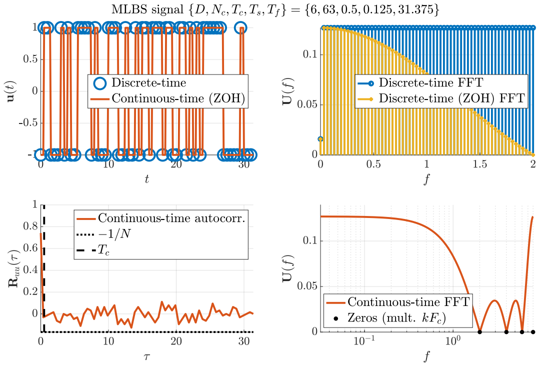
But, evaluate

- sensor noise, bandwidth, range
- drift, offset...

A detailed identification example #1

Signals construction

```
% Generate exciting signal
FBND         = [0 Fs/4];
REV          = false;
SHOW         = true;
[u1,t1,i1]   = insapack.mlbs(Ns,Ts,FBND,REV,SHOW);
```



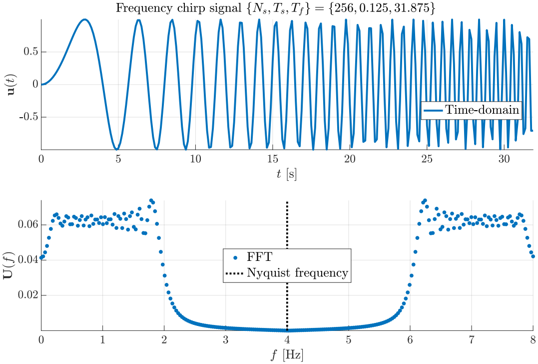
MLBS

- select a frequency range satisfying $f_{max} < f_s/2$
- check spectrum and analyse the zero notches

A detailed identification example #1

Signals construction

```
% Generate exciting signal
TYPE        = 'linear';% 'logarithmic';
[u2,t2,i2]  = insapack.chirp(Ns,Ts,FBND,REV,TYPE,SHOW);
```



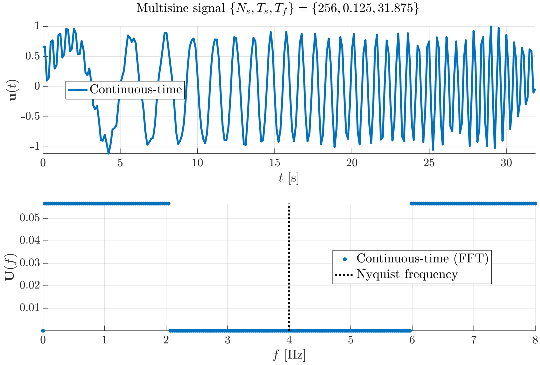
Chirp

- select a frequency range satisfying $f_{max} < f_s/2$
- check spectrum and low energy ranges

A detailed identification example #1

Signals construction

```
% Generate exciting signal
RPHI          = false;
ODD           = 'all';
[u3,t3,i3]    = insapack.multisine(Ns,Ts,FBND,RPHI,ODD,REV,SHOW);
```

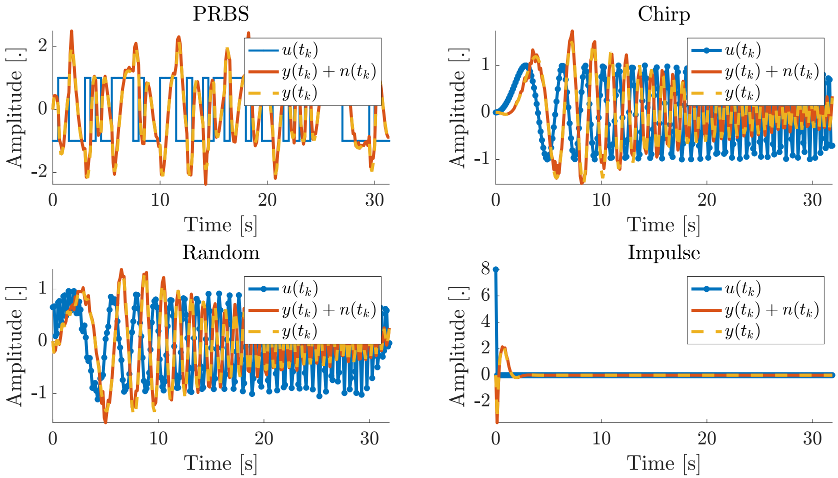


Multi-sine

- select a frequency range satisfying $f_{max} < f_s/2$
- check spectrum and low energy ranges

A detailed identification example #1

Signals construction (application on the system with noise)



A detailed identification example #1

Learning vs. validation data

```
nx      = 3;
u_val   = sig1.u;
t_val   = sig1.t;
```

```
sig     = sig3;
Ts      = sig.Ts;
t       = sig.t;
u       = sig.u;
y       = sig.y;
yn      = sig.yn;
ytrue   = lsim(G,u_val,t_val);
w       = logspace(-2,log10(1/Ts),200)*2*pi;
Gtrue   = freqresp(G,w);
```

Validation

- here we use MLBS as validation
- in practice use more !
- used metric on accuracy, eigenvalues, ...

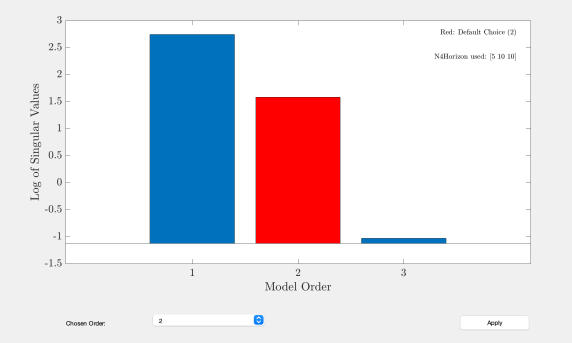
Learning

- here we use multi-sine
- can use more than one single
- can use averaged model
- can average multiple models...

A detailed identification example #1

Time-domain identification with Subspaces Framework

```
%%% IDENTIFICATION VIA N4SID
Hn4sid = n4sid(u,yn,1:nx,'Ts',Ts);
Hn4sid = stabsep(ss(Hn4sid));
yn4sid = lsim(Hn4sid,u_val,t_val);
Gn4sid = freqresp(Hn4sid,w);
```



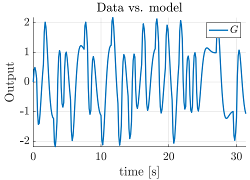
N4SID

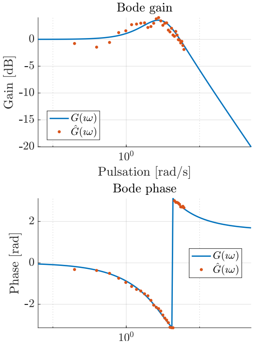
- can input a range of dimensions
- Hankel singular values are here to help selection
- Here we keep $n = 3$

A detailed identification example #1

Frequency-domain identification in the Loewner Framework

```
% Compute FRF
[f,U0,Y0,G0]    = insapack.non_param_freq(u,yn,Ts);
puls            = 2*pi*f(f<FBND(end)/2);
wRange          = 1:floor(length(puls)/2)*2;
puls            = 2*pi*f(wRange);
G0              = G0(wRange);
```





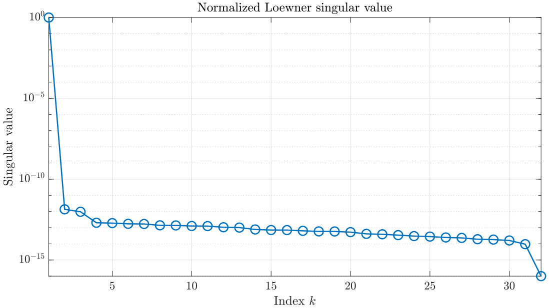
Compute FRF

- Apply non parametric analysis
- Estimate the frequency response / sigma (if multiple tests)
- Pre-treat the frequency data to remove the signals outside the excitation range, filter, ...
- Post-treat (stability separation)

A detailed identification example #1

Frequency-domain identification in the Loewner Framework

```
% Loewner
[la,mu,W,V,R,L] = insapack.data2loewner(puls,G0);
opt.target      = nx;
[hr,info]       = insapack.loewner_tng(la,mu,W,V,R,L,opt);
Hloe            = dss(info.Ar,info.Br,info.Cr,info.Dr,info.Er);
Hloe            = stabsep(Hloe);
yloe            = lsim(Hloe,u_val,t_val);
Gloe            = freqresp(Hloe,w);
```

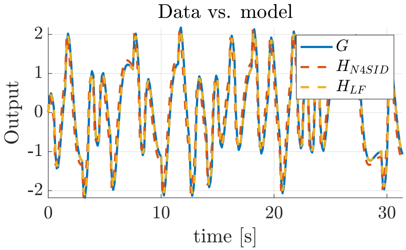


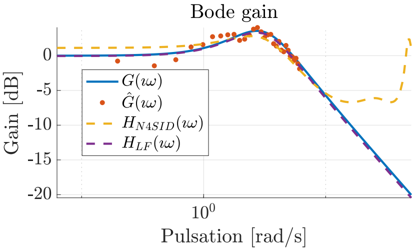
Loewner Framework

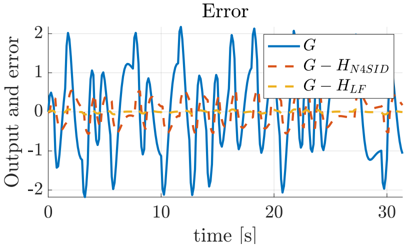
- Construct row and column data
- Apply Loewner and check Loewner singular decay
- Select the order
- Post-treat (stability separation)

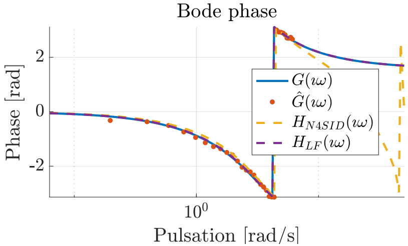
A detailed identification example #1

Comparison and validation







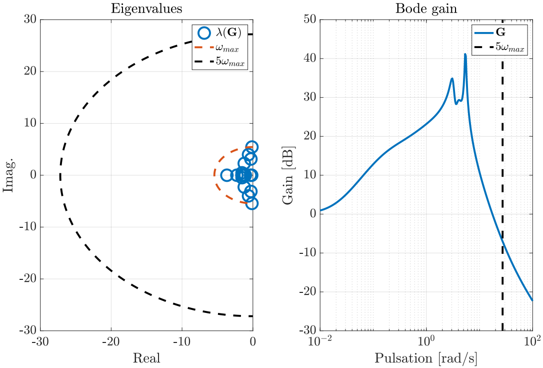


Content

A detailed identification example #2

Setting

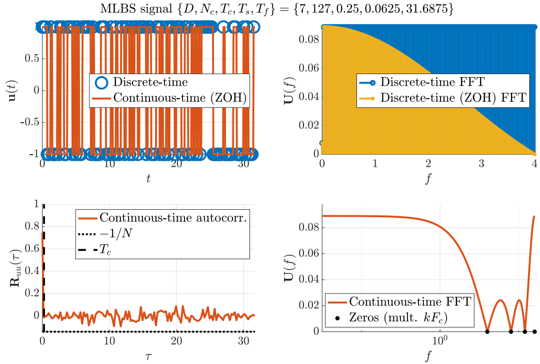
```
rng(1);
G       = stabsep(rss(20,1,1)); G = G/dcgain(G);
Ns      = 2^9;
fmax    = 5*max(abs(eig(G)))/2/pi;
Fs      = 2^(nextpow2(fmax)+1);
Ts      = 1/Fs;
```



A detailed identification example #2

Signals construction

```
% Generate exciting signal
FBND        = [0 Fs/4];
REV         = false;
SHOW        = true;
[u1,t1,i1]  = insapack.mlbs(Ns,Ts,FBND,REV,SHOW);
```



A detailed identification example #2

Signals construction

```
% Generate exciting signal
TYPE           = 'linear';% 'logarithmic';
[u2,t2,i2]     = insapack.chirp(Ns,Ts,FBND,REV,TYPE,SHOW);
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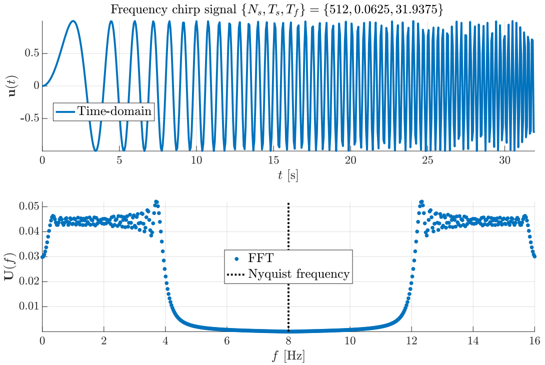

A detailed identification example #2

Signals construction

```
% Generate exciting signal
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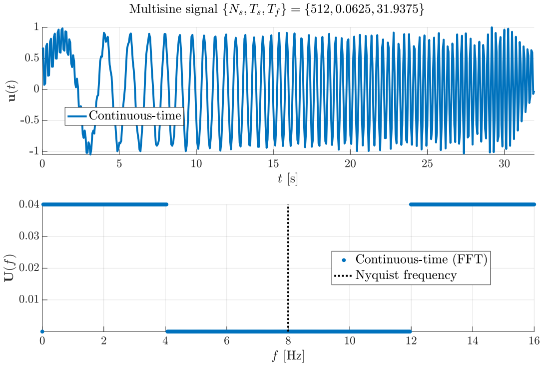
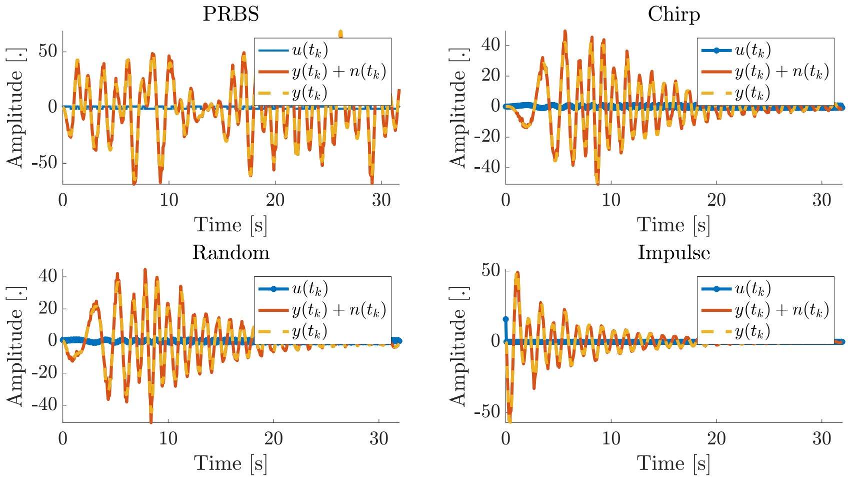


A detailed identification example #2

Signals construction



A detailed identification example #2

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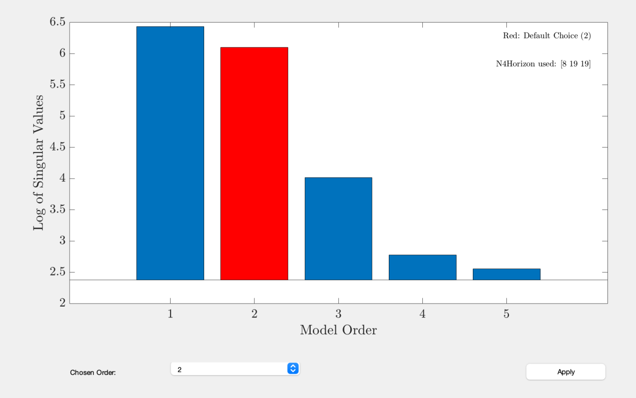
```



A detailed identification example #2

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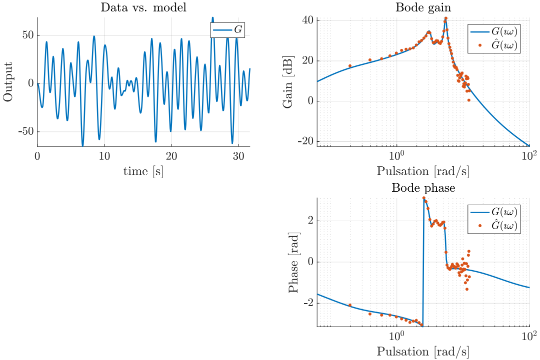

A detailed identification example #2

Frequency-domain identification in the Loewner Framework

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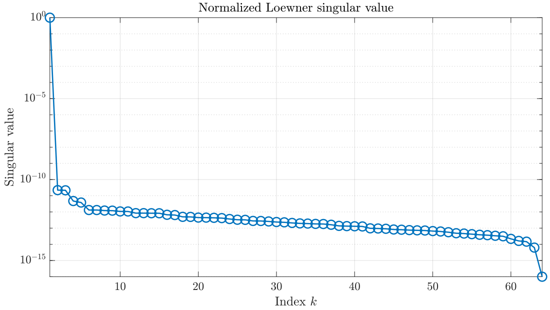
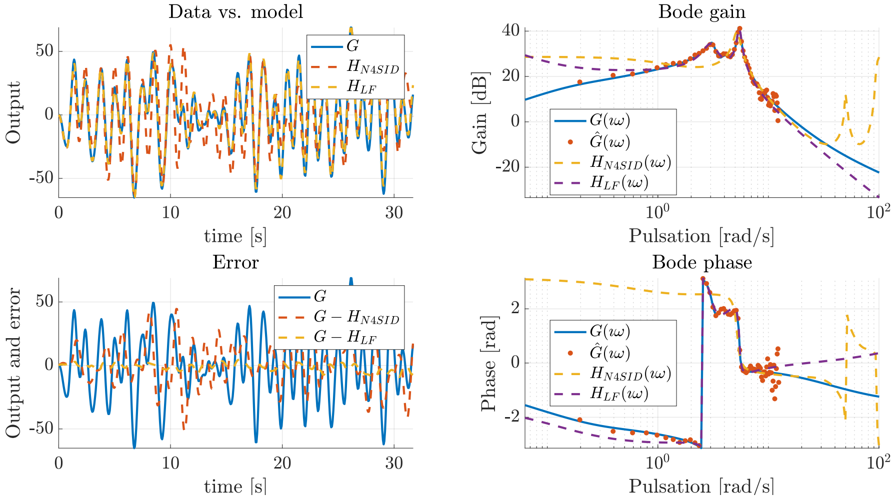


A detailed identification example #2

Comparison and validation



Content

Experimental setup

The experiments more in details

- DC motor
- Flying ping pong ball
- Small wind tunnel
- Water tanks
- Ball on a rail
- Temperature & flow

Experimental setup

The experiments more in details

- **DC motor**
- **Flying ping pong ball**
- **Small wind tunnel**
- **Water tanks**
- **Ball on a rail**
- **Temperature & flow**

- SISO, stable, linear
- SIMO, unstable, linear
  sensor potentially nonlinear
- SIMO, stable, linear & delayed
  sensor quite noisy
- SISO, unstable, linear & non-linear
  very slow
- SIMO, unstable, linear
  closed-loop identification
- MIMO, linear & switched
  possibly 2 identifications, 2 laws

Experimental setup

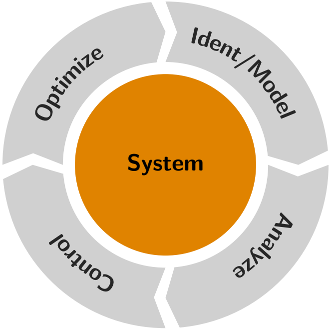
The experiments more in details

- **DC motor**
  - $\mathbf{u}$: DC voltage / $\mathbf{y}$: rotation speed
    objective: tracking, disturbance rejection, digital
- **Flying ping pong ball**
  - $\mathbf{u}$: DC voltage / $\mathbf{y}$: rotation speed & ball height
    objective: stabilization, disturbance rejection
- **Small wind tunnel**
  - $\mathbf{u}$: DC voltage / $\mathbf{y}$: pressure & flow
    objective: tracking, disturbance rejection
- **Water tanks**
  - $\mathbf{u}$: flow voltage / $\mathbf{y}$: water height
    objective: tracking, adaptive
- **Ball on a rail**
  - $\mathbf{u}$: DC voltage / $\mathbf{y}$: beam angle & ball position
    objective: stabilization, disturbance rejection
- **Temperature & flow**
  - $\mathbf{u}$: voltage / $\mathbf{y}$: rotation speed
    objective: tracking, switching (positive system)

Content

Working methodology

#1 identification and modeling

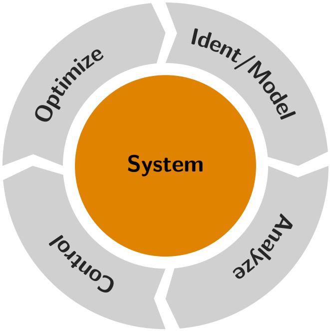


Some steps

- Analyze "visually" the system
- Ask yourself about its properties
- Build excitation & validation signals
- Chose a model structure / complexity
- Model black/grey/white box model

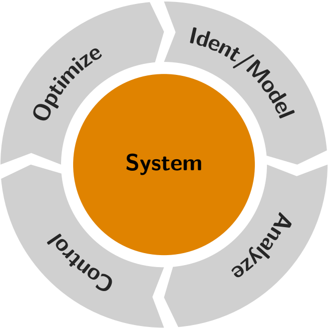
Working methodology

#2 analyze



Some steps

- Spectral: poles and zeros for multiple draw
- Time-domain: impulse, step, MLBS
- Frequency-domain: Bode gain & phase, Nyquist
- Parametric sensitivity
- Analyze realization complexity, parameters
- Construct a white(grey)-box equivalent model if possible
- Evaluate which variables are of interest? Gain, phase, delay, oscillation, ...



Working methodology

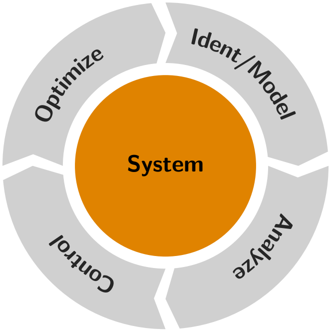
#3 design a control law

Quelques étapes

- Define control and performance objectives & constraints (saturation, velocity, bandwidth)
- Chose actuator & sensors to use (are they reliable?)
- Chose a control law structure
- Optimize the parameters of the control (using pole placement, time- or frequency-domain methods, norms...)
- Is an estimator useful?
- Analyze stability and robustness margins

Working methodology

#4 optimize the setup



Quelques étapes

- Filtering can be useful?
- How it behaves with discretization, sampling, delay, jitter
- Rounding gains, measurements
- Imagine sensor loss
- How to render the system more efficient, less expensive...

Content

Evaluation

Organization and evaluation

Organisation

- Group of 3 persons
- 5 sessions
  2 for identification
  1 for analysis
  1 for control
  1 for improvement & presentation
- Last session $\approx$ 2h+1h15 (all groups)

Evaluation

- Presentation
  10 minutes (slides & demonstration)
  5 minutes (questions)
- Notation:
  10 pts (presentation)
  10 pts (participation)
  Possible different notations according to engagement

Linear dynamical system identification

... basic elements and Labs guidelines

Charles Poussot-Vassal
February 20, 2026

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