Data-driven parametric reduced-order model for aircraft flutter monitoring (& control)

(in collaboration with Airbus)

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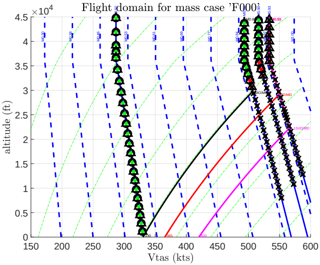
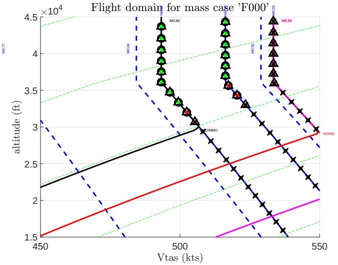
Aircraft flutter

- Coupling between structural dynamics and aerodynamics
- Responsible for stability loss and potential structural damage





Aircraft flutter equation

$$(s^2 M + sD + K)x = Q(s,\theta)x + Bu \text{ and } y = Cx,$$

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$$(s^2 M + sD + K)x = Q(s,\theta)x + Bu \text{ and } y = Cx,$$

- structural dynamics $M, D, K \in \mathbb{R}^{n \times n}$ $\Longrightarrow$ Known
- actuators $B \in \mathbb{R}^{n \times n_u}$ sensors $C \in \mathbb{R}^{n_y \times n}$ $\Longrightarrow$ Known
- generalized aeroelastic forces $Q(s,\theta) \in \mathbb{C}^{n \times n}$ $\Longrightarrow$ Discrete knowledge

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Generalized forces are partially known

$$Q(s_i, \theta_j) \text{ for } i = 1, \ldots, N,\ j = 1, \ldots, M$$

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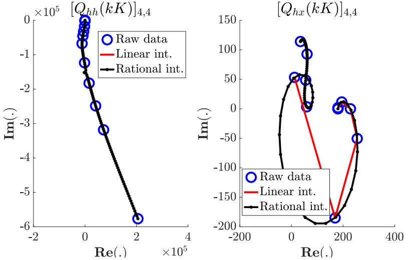

Complexity

- $\approx$ 100 states
- $\approx$ 100 outputs
- $\approx$ 10 inputs
- $Q(\imath\omega_i, \theta_j)$, $\{\omega_i\}_1^{10}$, $\{\theta_j\}_{j=1}^{30}$

Aircraft flutter objective and challenges are

- to evaluate the flutter occurrence over the frequency/parameter couple,
- to prevent it by monitoring the critical damping value, and
- to prevent its apparition via feedback control through control surfaces.

Our philosophy

$$(s^2M + sD + K)x = Q(s_i, \theta_j)x + Bu \text{ and } y = Cx,$$

- (Rectangular) MIMO pROM in the Loewner framework
- Data- and model-driven detection algorithms
- Robust control/analysis using $\mathcal{H}_\infty$ and μ techniques

Content

Flutter data-driven pROM

Linear Loewner realization

Given $\{\mu_i, \mathbf{v}_i\}$, $\{\lambda_j, \mathbf{w}_j\}$ data, seek $\mathbf{H}$, s.t.

$$\begin{aligned} \mathbf{H}(\mu_i) &= \mathbf{v}_i \\ \mathbf{H}(\lambda_j) &= \mathbf{w}_j \end{aligned}$$

$i = 1, \ldots, q;\ j = 1, \ldots, k.$

Rational interpolation
$\mathbf{H}(s) = \mathbf{W}(-s\mathbb{L} + \mathbb{M})^{-1}\mathbf{V}$

- underlying rational (r) order

$$\begin{aligned} r &= \mathbf{rank}(\xi\mathbb{L} - \mathbb{M}) \\ &= \mathbf{rank}([\mathbb{L}, \mathbb{M}]) \\ &= \mathbf{rank}([\mathbb{L}^H, \mathbb{M}^H]^H) \end{aligned}$$

- and McMillan (ν) order

$$\nu = \mathbf{rank}(\mathbb{L})$$

- $\mathbb{L}$ a d $\mathbb{M}$ satisfy specific Sylvester equations
- Both $\mathbb{L}$ and $\mathbb{M}$ are input-output independents

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A.C. Antoulas, S. Lefteriu and A.C. Ionita, *"Ch. 8: A tutorial to the Loewner framework for model reduction"*, Model Reduction and Approximation.

A.J. Mayo and A.C. Antoulas, *"A framework for the solution of the generalized realisation problem"*, Linear Algebra and its Applications, 425(2-3), 2007.

Flutter data-driven pROM

Linear Loewner barycentric form

Given $\{\mu_i, \mathbf{v}_i\}$, $\{\lambda_j, \mathbf{w}_j\}$ data, seek $\mathbf{H}$, s.t.

$$\begin{array}{rcl} \mathbf{H}(\mu_i) & = & \mathbf{v}_i \\ \mathbf{H}(\lambda_j) & = & \mathbf{w}_j \end{array}$$

$i = 1, \ldots, q$; $j = 1, \ldots, k$.

Rational interpolation
$\mathbf{H}(s) = C\mathbf{\Phi}(s)^{-1}B$

- Non-minimal realization order and McMillan degree

Where $\mathbf{H}$ in Lagrangian basis $\mathbb{L}\mathbf{c} = 0$

$$\mathbf{H}(s) = \underbrace{\mathbf{cw}}_{C} \underbrace{\begin{bmatrix} \mathbf{L}_{s,\lambda,k} \\ \mathbf{c} \end{bmatrix}^{-1}}_{\mathbf{\Phi}(s)^{-1}} \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B}$$

$$\mathbf{L}_{s,\lambda,k} = \begin{bmatrix} s - \lambda_1 & \lambda_2 - s & \\ s - \lambda_1 & & \lambda_3 - s \\ \vdots & & & \ddots \end{bmatrix} \in \mathbb{C}^{(k-1)\times k}$$

A.C. Antoulas, S. Lefteriu and A.C. Ionita, *"Ch. 8: A tutorial to the Loewner framework for model reductop,"*, Model Reduction and Approximation.

Flutter data-driven pROM

Linear parametric Loewner barycentric form

Parametric interpolation problem

Given the data (λ_i, μ_k, π_j and ν_l are distinct):

$$
\begin{aligned}
[s_1, \ldots, s_N] &= [\lambda_1, \ldots, \lambda_{\overline{n}}] \cup [\mu_1, \ldots, \mu_{\underline{n}}] \\
[\theta_1, \ldots, \theta_M] &= [\pi_1, \ldots, \pi_{\overline{m}}] \cup [\nu_1, \ldots, \nu_{\underline{m}}] \\
\mathbf{\Phi} &= \left[\begin{array}{c|c} \mathbf{w}_{ij} & \mathbf{\Phi}_{12} \\ \hline \mathbf{\Phi}_{21} & \mathbf{v}_{kl} \end{array}\right]
\end{aligned}
$$

we seek $\mathbf{H}(s, \theta) = C(\theta)(sE - A(\theta))^{-1} B(\theta)$ s.t.

$$
\begin{aligned}
\mathbf{H}(\lambda_i, \pi_j) &= \mathbf{w}_{ij} \quad i = 1 \ldots \overline{n} \text{ and } j = 1, \ldots, \overline{m} \\
\mathbf{H}(\mu_k, \nu_l) &= \mathbf{v}_{kl}^T \quad k = 1 \ldots \underline{n} \text{ and } l = 1, \ldots, \underline{m}
\end{aligned}
$$

A.C. Ionita and A.C. Antoulas, *"Data-Driven Parametrized Model Reduction in the Loewner Framework"*, SIAM on Scientific Computing, Vol 36(3).

Flutter data-driven pROM

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$$[s_1, \dots, s_N] = [\lambda_1, \dots, \lambda_{\overline{n}}] \cup [\mu_1, \dots, \mu_{\underline{n}}]$$
$$[\theta_1, \dots, \theta_M] = [\pi_1, \dots, \pi_{\overline{m}}] \cup [\nu_1, \dots, \nu_{\underline{m}}]$$

$$\mathbf{\Phi} = \left[\begin{array}{c|c} \mathbf{w}_{ij} & \mathbf{\Phi}_{12} \\ \hline \mathbf{\Phi}_{21} & \mathbf{v}_{kl} \end{array}\right]$$

p-Loewner

$$[\mathbb{L}_2]_{i,j}^{k,l} = \frac{\mathbf{v}_{kl} - \mathbf{w}_{ij}}{(\mu_k - \lambda_i)(\nu_l - \pi_j)}$$
$$[\mathbb{L}_{\lambda_i}] = \text{Loewner of } i\text{th row of } \mathbf{\Phi}$$
$$[\mathbb{L}_{\pi_j}] = \text{Loewner of } j\text{th column of } \mathbf{\Phi}$$

$$\widehat{\mathbb{L}}_2 \mathbf{c} = \begin{bmatrix} \mathbb{L}_2 \\ \mathbb{L}_\lambda \\ \mathbb{L}_\pi \end{bmatrix} \mathbf{c} = 0$$

Rational parametric model

$$\mathbf{H}(s,\theta) = \frac{\sum_{i=1}^{\overline{n}} \sum_{j=1}^{\overline{m}} \frac{\mathbf{c}_{ij}\mathbf{w}_{ij}}{(s-\lambda_i)(\theta-\pi_j)}}{\sum_{i=1}^{\overline{n}} \sum_{j=1}^{\overline{m}} \frac{\mathbf{c}_{ij}}{(s-\lambda_i)(\theta-\pi_j)}}$$

A.C. Ionita and A.C. Antoulas, *"Data-Driven Parametrized Model Reduction in the Loewner Framework"*, SIAM on Scientific Computing, Vol 36(3).

MIMO (right Coprime) realization

$$E = \begin{bmatrix} I_{n_u} & -I_{n_u} & & \\ \vdots & & \ddots & \\ I_{n_u} & & & -I_{n_u} \\ 0 & 0 & \dots & 0 \end{bmatrix} \text{ and } A = \begin{bmatrix} \lambda_1 I_{n_u} & -\lambda_2 I_{n_u} & & \\ \vdots & & \ddots & \\ \lambda_1 I_{n_u} & & & -\lambda_{\overline{n}} I_{n_u} \\ -\alpha_1(\theta) & -\alpha_2(\theta) & \dots & -\alpha_{\overline{n}}(\theta) \end{bmatrix}$$

$$C = \begin{bmatrix} \beta_1(\theta) & \beta_2(\theta) & \dots & \beta_{\overline{n}}(\theta) \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 & \dots & I_{n_u} \end{bmatrix}^T$$

where

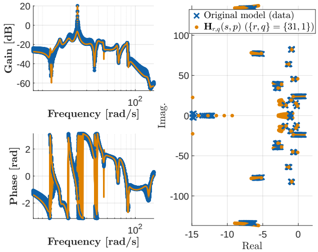
$$\alpha_i(\theta) = \sum_{j=1}^{\overline{m}} c_{ij} \mathbf{q}_j(\theta) \qquad \beta_i(\theta) = \sum_{j=1}^{\overline{m}} \mathbf{w}_{ij} c_{ij} \mathbf{q}_j(\theta) \qquad \mathbf{q}_j(\theta) = \prod_{j'=1, j' \neq i}^{\overline{m}} (\theta - \pi_{j'})$$

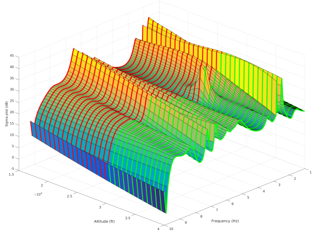
T. Vojkovic, D. Quero, C. P-V and P. Vuillemin, *"Low-Order Parametric State-Space Modeling of MIMO Systems in the Loewner Framework"*, submitted to SIAM.

Flutter data-driven pROM

Infer a parametric flutter model from data

$\mathbf{G}(s,\theta) = \mathbf{l}_b^T(\theta) C \left(s^2 M + sB + K - Q(s,\theta)\right)^{-1} B\mathbf{r}_b(\theta)$ where θ is the flight altitude.





J. Toledo et al., *"Flutter detection using reduced-order dynamic isolation observer"*, sub. IFAC WC.

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The rational parametric function that recovers both I/O transfer and its stability properties is

$$\mathbf{H}(s,\theta) = (C + C_1\theta)(sE - A - A_1\theta)^{-1}B$$

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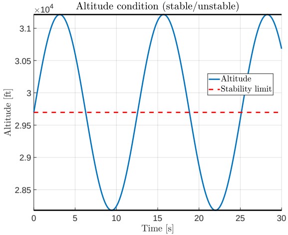

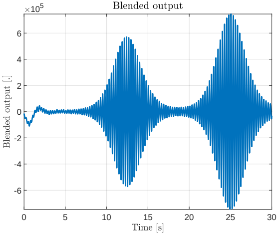

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Content

Flutter detection

Data-driven approach

Grounded on simulated data from

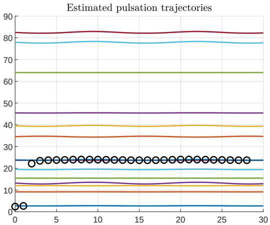
$$\mathbf{H}(s,\theta) = (C + C_1\theta)(sE - A - A_1\theta)^{-1}B$$

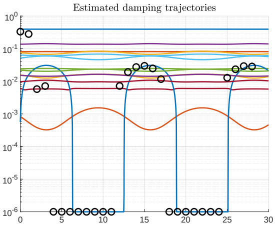
One collects online

$$\mathbf{Y} = [\mathbf{y}(t_1), \mathbf{y}(t_2), \ldots, \mathbf{y}(t_N)] \in \mathbb{R}^{1\times N}.$$

And generates the sampled-time model

$$E\mathbf{x}(t_k + h) = A\mathbf{x}(t_k)\ ,\ \mathbf{y}(t_k) = C\mathbf{x}(t_k)\ \text{et}\ E\mathbf{x}(t_1) = B$$





Flutter detection

Model-driven approach

Using the second-order-like model with blended input/output (dynamic isolation):

$$l^\top H(s) r \approx \frac{b_0}{s^2 + \theta s + a_0}$$

one can design a non-linear observer with an extended state vector:

$$\frac{d}{dt}\begin{pmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \\ \hat{\theta}(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 & -y(t) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \\ \hat{\theta}(t) \end{pmatrix} + \begin{pmatrix} 0 \\ b_0 u(t) - a_0 y(t) \\ 0 \end{pmatrix} - \begin{pmatrix} L_1(t) \\ L_2(t) \\ L_3(t) \end{pmatrix} (\hat{y}(t) - y(t))$$

where θ represents the damping of the system.

Main idea: the dynamics of the estimation error resembles a port-Hamiltonian system.

J. Toledo et al., *"Flutter detection using reduced-order dynamic isolation observer"*, sub. IFAC WC.

Model-driven approach

Simulation results: using a fixed θ for design, robustness test against slightly different ones

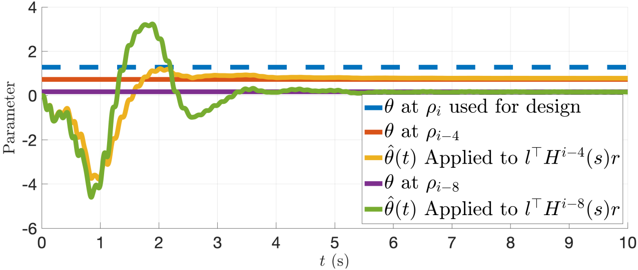
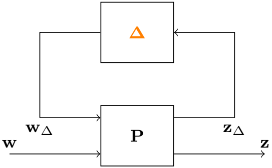


Figure: Damping for an altitude θ_i (blue), for an altitude θ_{i-4} (red) with its estimation (yellow), and damping for an altitude θ_{i-8} (violet) with its estimation (green).

Content

Flutter robust control

Design



LFT form: a open way to robust control

Many constraints: controller type, different frequency bands, stability certificates, etc.

Several tools available: **`musyn`**, **`hinfstruct`**, **`robuststab`**, ...

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Flutter robust control

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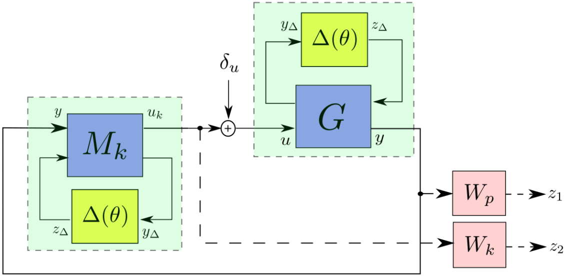

Design a *parametric, dynamic, output-feedback* controller such as

$$K(\theta) : \begin{cases} \dot{x}_c & = A_c(\theta)x_c + B_c(\theta)y \\ u & = C_c(\theta)x_c \end{cases}$$

where the parameter-dependent matrices are affine (*e.g.*, $A_c(\theta) = A_{c,0} + A_{c,1}\theta$).

Flutter robust control

Robust validation

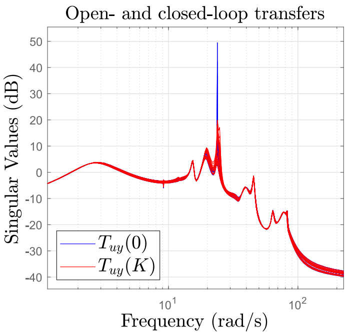

Figure: Open- and closed-loop transfers

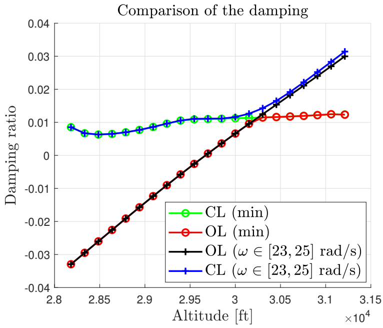

Figure: Open- and closed-loop damping ratios

Robustness analysis: $\mu \in [0.5699,\ 0.9390] \to$ robust stability against all considered θ.

Content

Conclusions

- Flutter parametric model construction (via LF)
- Flutter parametric model allows time-domain
- Link with uncertain model
- Link with robust control

  $\rightarrow$ direct impact in engineers life

... still so much to do

Issues

- we still don't known were the exact instability is $\rightarrow$ verification against discrete data points
- ack, work in collaboration with Airbus

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