Civil aircraft reduced order modelling, control and validation: Numerical and experimental challenges

... how much dynamical data science is a strong ally?

Charles Poussot-Vassal
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(e-) St Raphael, France
FrPL3 Plenary Session, ROOM SR1

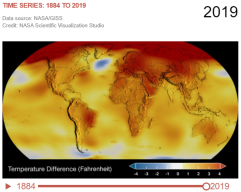
Clean Sky ONERA THE FRENCH AEROSPACE LAB IEEE Control Systems Society™

Introduction

Mobility and civil aviation challenges

(Aircraft) mobility plays a central role...

in our **life style** (leisure travels) and **societal organisation** (commercial exchanges)
but suffers of
severe **environmental changes** (global warming).

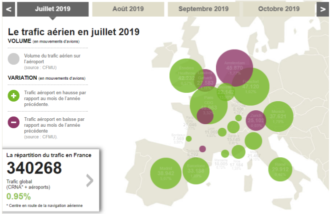


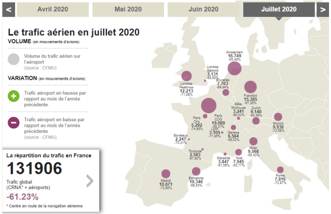
Left : world view of civil aviation (August 19th, 2020)
[https://www.flightradar24.com/]

Right : 2019 temperature differences wrt. 1880
[https://climate.nasa.gov/vital-signs/global-temperature/]

(Aircraft) mobility plays a central role...

in our **life style** (leisure travels) and **societal organisation** (commercial exchanges)
but suffers of
severe **environmental changes** (global warming) and **pandemic issues** (COVID19).





Civil aircraft traffic in the main French and border airports (July, 2019 vs. 2020)
[http://salledelecture-ext.aviation-civile.gouv.fr/externe/mouvementsDavions/index.php]

Introduction

Some civil aircraft challenges

Reducing footprint and emissions while remaining competitive?

- Enhance aviation flow (traffic optimisation)
- Enhance motors technology (combustion, electricity)
- **Enhance aircraft efficiency (weight, structure)**

Dynamical data science...

Dynamical systems, **Linear algebra** and **Computational science** provide an appropriate toolkit to address these problems, make decisions, enhance the development process and **optimise aircraft systems, in a certified and competitive industrial context**.

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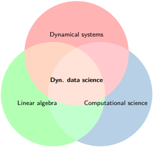

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Dynamical systems, **Linear algebra** and **Computational science** provide an appropriate toolkit to address these problems, make decisions, enhance the development process and **optimise aircraft systems, in a certified and competitive industrial context**.

Introduction

Today's talk outline

How to enhance aircraft efficiency and design process?
Use dynamical data science

Application on a **Generic** Business Jet aircraft **models and experiments**
Gust load alleviation and **Vibration reduction**

1. Aircraft gust load and vibration **control problems**
2. Aircraft large-scale **modelling**, **model approximation** and **reduction**
3. Aircraft gust load and vibration **control design** in a industrial setup
4. Aircraft **control implementation** and **uncertainty analysis**
5. Aircraft ground and flight **experimental validations** and **data-driven modelling**

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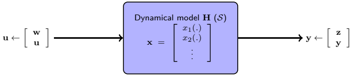
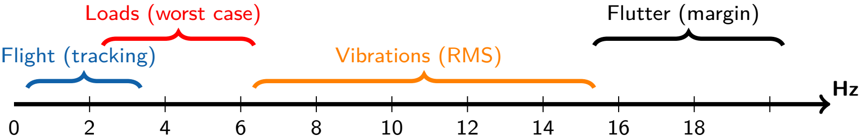


Aircraft control system problem

Sequential feedback control (industrial) problem description and architecture



(value ranges are indicative and may vary from an aircraft to an other)

Aircraft control system problem

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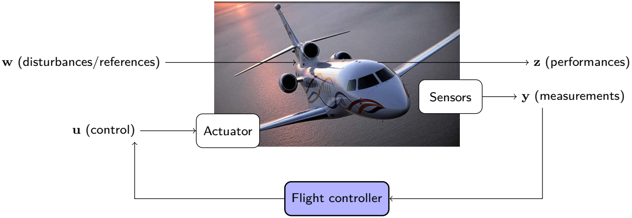

Flight qualities impact handling $\Rightarrow$ aircraft manoeuvrability

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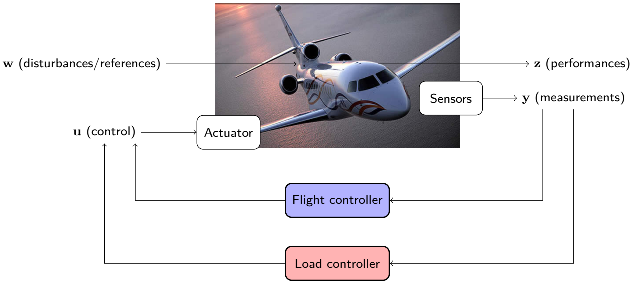

Flight qualities impact handling $\Rightarrow$ aircraft manoeuvrability
Loads impact structure $\Rightarrow$ aircraft weight reduction

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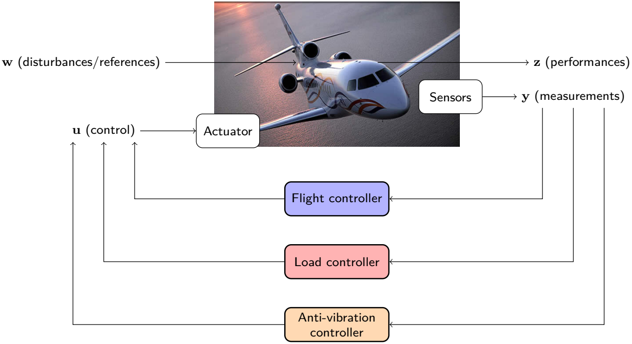

Flight qualities impact handling ⇒ aircraft manoeuvrability
Loads impact structure ⇒ aircraft weight reduction
Vibrations impact fatigue ⇒ aircraft life-time

Aircraft control system problem

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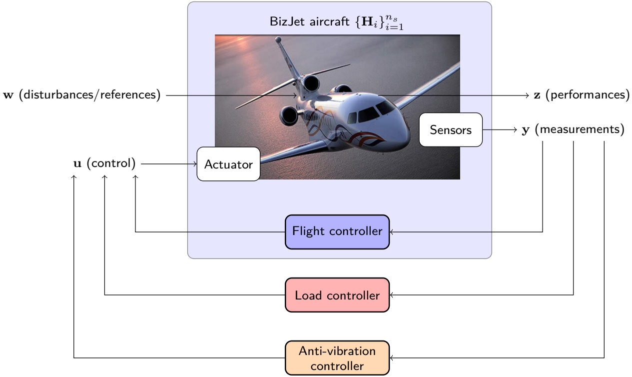

Aircraft are dynamical systems

How to **model**, **design** controllers, **analyse** and **validate** ?
Today: focus on **Gust Load Alleviation** and **Vibration Reduction** objectives

Aircraft control system problem

Find solutions fitting the industrial process, constraints and philosophy

Dynamical model → Optimisation, control, ... → A/C implementation and tests → Tests exploitation

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C. Meyer, J. Prodigue, G. Broux, O. Cantinaud and C. P-V., *"Ground test for vibration control demonstrator"*, in Proceedings of the International Conference on Motion and Vibration Control (MOVIC), Southampton, United Kingdom, July, 2016, pp. 1-12.

D. Quero-Martin, P. Vuillemin and C. P-V., *"A Generalized State-Space Aeroservoelastic Model based on Tangential Interpolation"*, in Aerospace, Special Issue Aeroelasticity, January 2019 (Open Access paper).

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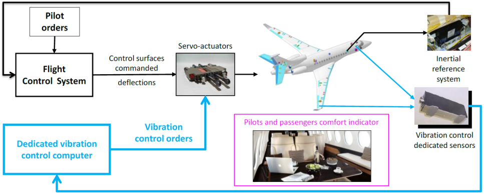

Hidden constraints

Find solution that fit the industrial workflow.

Sensors/Actuators

Use existing control layout.

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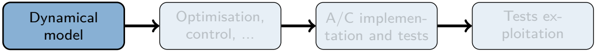

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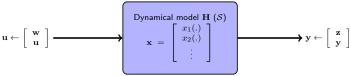
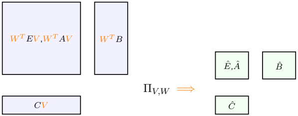
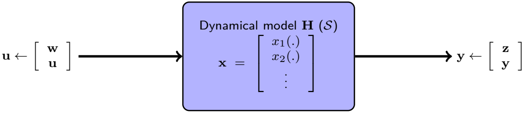




Aircraft reduced order modelling

A dynamical model H is a function mapping input u to output y signals of a system Σ



$$\mathcal{S} : \begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}\Big(\mathbf{x}(t), \mathbf{u}(t)\Big) & \text{, differential equation} \\ 0 = \mathbf{g}\Big(\mathbf{x}(t), \mathbf{u}(t)\Big) & \text{, algebraic equation} \\ \mathbf{y}(t) = \mathbf{h}\Big(\mathbf{x}(t), \mathbf{u}(t)\Big) & \text{, output equation} \end{cases}$$

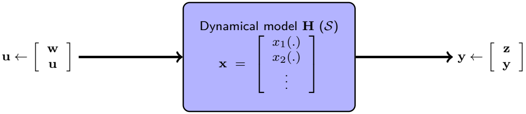
Nonlinear models

Mathematical and numerical tools **not simple** to deploy in industrial context

Aircraft reduced order modelling

System and linear models

A linear dynamical model $\mathbf{H}$ is a linear function mapping input $\mathbf{u}$ to output $\mathbf{y}$ signals of a system Σ



$$\mathcal{S} : \begin{cases} E\dot{\mathbf{x}}(t) &= \mathbf{A}\Big(\mathbf{x}(t)\Big) + B\mathbf{u}(t) & \text{, algebraic - differential equation} \\ \mathbf{y}(t) &= C(t) & \text{, output equation} \end{cases}$$

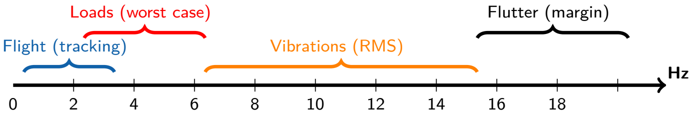
$$\begin{array}{ccl} \mathbf{A}\Big(\mathbf{x}(t)\Big) & = & A\mathbf{x}(t) \text{ or } A\mathbf{x}(t) + A_\tau \mathbf{x}(t-\tau) \text{ ...} \\ E & & \text{possibly singular} \end{array}$$

Linear models

Mathematical and numerical tools
ready to deploy in industrial context

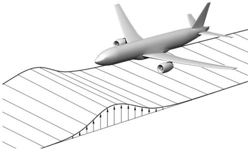
Aircraft reduced order modelling

Models tailored to phenomena and problems specificities



#1 gust load control models

- Multiple flight config. ($\approx \#1,000$)
- Large-scale (≈ 300 internal variables)
- Polynomial (due to pos./vel./accel. inputs)
- Irrational (due to internal delays)
- Band-limited phenomena $[2\ 6]$Hz



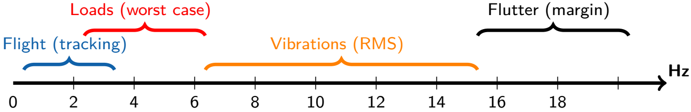
P. Lancelot et al., "Investigation of the unsteady flow over a wing gust excitation", IFASD 2017.

Models / data of the form

$$\mathbf{H}(s) = C\left(sE - A_0 - A_1 e^{-\tau_1 s} - A_2 e^{-\tau_2 s}\right)^{-1} B$$

Aircraft reduced order modelling

Models tailored to phenomena and problems specificities



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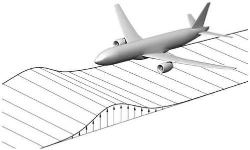

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Aircraft reduced order modelling

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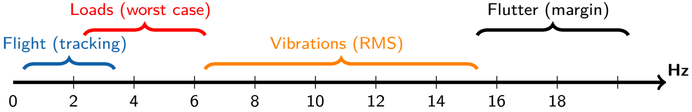


#2 vibration control models

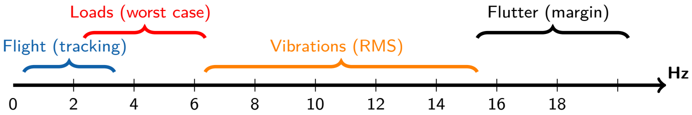
- Multiple flight config. ($\approx \#100$)
- Large-scale (≈ 800 internal variables)
- Rational (no polynomial part)
- Band-limited phenomena $[6\ 15]$Hz
- Cross validation using data in GVT

Models / data of the form

$\mathbf{H}(s) = C\,(sI - A)^{-1}\,B$ (and $\{\mathbf{u}(t_k), \mathbf{y}(t_k)\}$ for tests)

Aircraft reduced order modelling

Models tailored to phenomena and problems specificities



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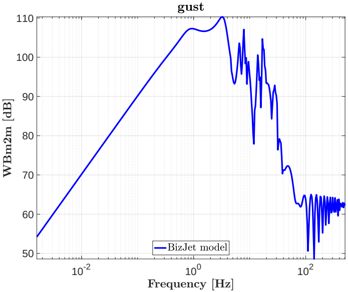
Aircraft reduced order modelling

Models tailored to phenomena and problems specificities (the gust load case #1)

Large aeroelastic models

- $\approx$ 300 state variables
- Irrational and polynomial
- Not suitable for analysis and control



$$\mathbf{H}(s) = C\left(sE - A_0 - A_1 e^{\tau_1 s} - A_2 e^{\tau_2 s}\right)^{-1} B$$

$$\mathbf{H}(s) = \mathbf{H}_{\text{ODE irrat.}} + \mathbf{P}_{\text{Poly.}}$$

Aircraft reduced order modelling

Models tailored to phenomena and problems specificities (the gust load case #1)

Large aeroelastic models

- $\approx$ 300 state variables
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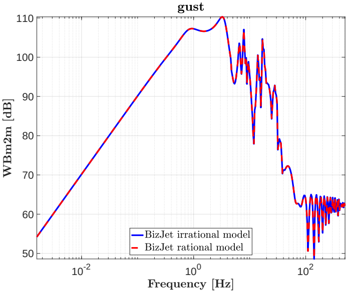
- Not suitable for analysis and control

$\Longrightarrow$

Model Approximation

Small and accurate models

- $\approx$ 400 state variables
- Rational (and polynomial)
- Suitable for analysis but not for control



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$\Downarrow$ (Rational approximation)

$$\mathbf{H}(s) = C(sE - A)^{-1} B.$$

Aircraft reduced order modelling

Models tailored to phenomena and problems specificities (the gust load case #1)

Large aeroelastic models

- $\approx$ 300 state variables
- Irrational and polynomial
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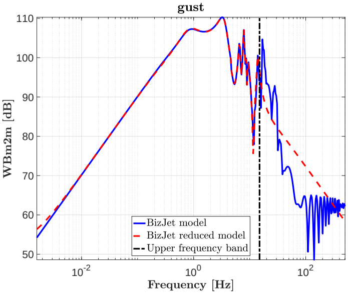
$\Longrightarrow$

Model Order Reduction

Small and accurate models

- $\approx$ 30 state variables
- Rational (over frequency range)
- Suitable for analysis and control



$$\mathbf{H}(s) = C\left(sE - A_0 - A_1 e^{\tau_1 s} - A_2 e^{\tau_2 s}\right)^{-1} B$$

$$\mathbf{H}(s) = \mathbf{H}_{\text{ODE irrat.}} + \mathbf{P}_{\text{Poly.}}$$

$\Downarrow$ (Rational approximation)

$$\mathbf{H}(s) = C(sE - A)^{-1}B.$$

$\Downarrow$ (Order reduction)

$$\hat{\mathbf{H}}(s) = \hat{C}(s\hat{E} - \hat{A})^{-1}\hat{B}.$$

Aircraft reduced order modelling

Focus on model approximation and reduction

Let us consider $\mathbf{H}$, a n_u inputs, n_y outputs linear dynamical system (evaluation) described by the **complex-valued function from $\mathbf{u}$ to $\mathbf{y}$, of order n**[1]

$$\mathbf{H} : \mathbb{C} \rightarrow \mathbb{C}^{n_y \times n_u}$$

model approximation consists in finding $\hat{\mathbf{H}}$, mapping $\mathbf{u}$ to $\hat{\mathbf{y}}$, of order $r \ll n$

$$\hat{\mathbf{H}} : \mathbb{C} \rightarrow \mathbb{C}^{n_y \times n_u},$$

that **well reproduces the input-output behaviour**

[1] n large or ∞, or just data.

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that **well reproduces the input-output behaviour** and equipped with realisation *e.g.*

$$\hat{\mathbf{H}} = \hat{C} \left(s\hat{E} - \hat{A} \right)^{-1} \hat{B}$$

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$$\hat{\mathbf{H}} = \hat{C}\left(s\hat{E} - \hat{A}\right)^{-1}\hat{B}$$

"Well reproduce..."? : $\hat{\mathbf{H}}$ is a "good" approximation of $\mathbf{H}$ if for any driving $\mathbf{u}(t)$, $(\mathbf{H} - \hat{\mathbf{H}})\mathbf{u}(t) = \mathbf{y}(t) - \hat{\mathbf{y}}(t)$ is "small"

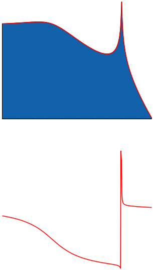
[1] n large or ∞, or just data.

Aircraft reduced order modelling

Focus on model approximation and reduction ($\mathcal{H}_2$ problem)

$\mathcal{H}_2$ model approximation

$$\hat{\mathbf{H}} := \arg \min_{\substack{\mathbf{G} \in \mathcal{H}_2 \\ \mathbf{rank}(\mathbf{G}) = r \ll n}} ||\mathbf{H} - \mathbf{G}||_{\mathcal{H}_2}$$



$\mathcal{H}_2$-norm of $\mathbf{H} - \hat{\mathbf{H}}$

Frequency-domain interpretation:

$$||\mathbf{H}||^2_{\mathcal{H}_2} := \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{tr}\left(\overline{\mathbf{H}(\imath\nu)}\mathbf{H}^T(\imath\nu)\right) d\nu$$

Time-domain interpretation:

$$||\mathbf{y}(t) - \hat{\mathbf{y}}(t)||_{L_\infty} \leq ||\mathbf{H} - \hat{\mathbf{H}}||_{\mathcal{H}_2} ||\mathbf{u}(t)||_{L_2}$$

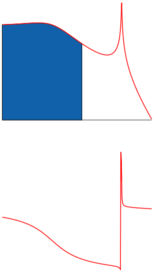
S. Gugercin and A C. Antoulas and C A. Beattie, *"$\mathcal{H}_2$ Model Reduction for Large Scale Linear Dynamical Systems"*, SIAM Journal on Matrix Analysis and Applications, 2008, Vol. 30(2), pp. 609-638.

Aircraft reduced order modelling

Focus on model approximation and reduction ($\mathcal{H}_{2,\Omega}$ problem)

$\mathcal{H}_{2,\Omega}$ model approximation

$$\hat{\mathbf{H}} := \arg \min_{\substack{\mathbf{G} \in \mathcal{H}_\infty \\ \mathbf{rank}(\mathbf{G}) = r \ll n}} ||\mathbf{H} - \mathbf{G}||_{\mathcal{H}_{2,\Omega}}$$



$\mathcal{H}_{2,\Omega}$-norm of $\mathbf{H} - \hat{\mathbf{H}}$

(finite support) Frequency-domain interpretation:

$$||\mathbf{H}||^2_{\mathcal{H}_{2,\Omega}} := \frac{1}{\pi} \int_\Omega \mathbf{tr}\left(\overline{\mathbf{H}(\imath\nu)}\mathbf{H}^T(\imath\nu)\right) d\nu$$

P. Vuillemin, C. P-V. and D. Alazard, *"$\mathcal{H}_2$ optimal and frequency-limited approximation methods for large-scale LTI dynamical systems"*, in Proc. of the IFAC SSSC, Grenoble, France, 2013, pp. 719-724.

Aircraft reduced order modelling

Focus on model approximation and reduction by interpolation ($\mathcal{H}_2$)

If $\hat{\mathbf{H}}$ is solution of the $\mathcal{H}_2$ approximation problem, then

$$\begin{array}{rcl} \mathbf{H}(-\hat{\lambda}_l) & = & \hat{\mathbf{H}}(-\hat{\lambda}_l) \\ \mathbf{H}'(-\hat{\lambda}_l) & = & \hat{\mathbf{H}}'(-\hat{\lambda}_l) \end{array}$$

where $\hat{\lambda}_l := \mathbf{eig}(\hat{E}, \hat{A})^a$ and

$$\mathbf{H}(s) = \sum_{j=1}^{n} \frac{\phi_j}{s - \lambda_j}$$

[a] $l = 1, \ldots, r$

By considering

$$\begin{array}{rcl} \hat{\mathbf{H}}(s) & = & \displaystyle\sum_{k=1}^{r} \frac{\hat{\phi}_k}{s - \hat{\lambda}_k} \\ & = & \hat{\mathbf{c}} \left(s\hat{E} - \hat{A} \right)^{-1} \hat{\mathbf{b}} \end{array}$$

Stands as the central result $\Rightarrow$ but how to determine $\hat{\lambda}_l$?

S. Gugercin and A C. Antoulas and C A. Beattie, *"$\mathcal{H}_2$ Model Reduction for Large Scale Linear Dynamical Systems"*, SIAM Journal on Matrix Analysis and Applications, 2008, Vol. 30(2), 2008, pp. 609-638.

K. A. Gallivan, A. Vanderope, and P. Van-Dooren, *"Model reduction of MIMO systems via tangential interpolation"*, SIAM Journal of Matrix Analysis and Application, 2004, Vol. 26(2), pp. 328-349.

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where $\hat{\lambda}_l := \mathbf{eig}(\hat{E}, \hat{A})$[a] and

$$\mathbf{H}(s) = \sum_{j=1}^{n} \frac{\phi_j}{s - \lambda_j}$$

[a] $l = 1, \ldots, r$

If a realisation of $\mathbf{H}$ exists,

$$\mathbf{H}(s) = \mathbf{c}\,(sE - A)^{-1}\,\mathbf{b}$$

S. Gugercin and A C. Antoulas and C A. Beattie, *"$\mathcal{H}_2$ Model Reduction for Large Scale Linear Dynamical Systems"*, SIAM Journal on Matrix Analysis and Applications, 2008, Vol. 30(2), pp. 609-638.

Aircraft reduced order modelling

Focus on model approximation and reduction by interpolation ($\mathcal{H}_2$)

If $\hat{\mathbf{H}}$ is solution of the $\mathcal{H}_2$ approximation problem, then

$$\begin{array}{rcl} \mathbf{H}(-\hat{\lambda}_l) & = & \hat{\mathbf{H}}(-\hat{\lambda}_l) \\ \mathbf{H}'(-\hat{\lambda}_l) & = & \hat{\mathbf{H}}'(-\hat{\lambda}_l) \end{array}$$

where $\hat{\lambda}_l := \mathbf{eig}(\hat{E}, \hat{A})$[a] and

$$\mathbf{H}(s) = \sum_{j=1}^{n} \frac{\phi_j}{s - \lambda_j}$$

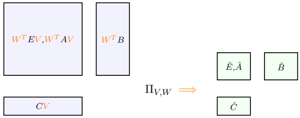
[a] $l = 1, \dots, r$

If a realisation of $\mathbf{H}$ exists,

$$\mathbf{H}(s) = \mathbf{c}\,(sE - A)^{-1}\,\mathbf{b}$$

$\Downarrow$ **Petrov-Galerkin (interp.)**

$$\hat{\mathbf{H}}(s) = \mathbf{c}V\left(sW^TEV - W^TAV\right)^{-1} W^T\mathbf{b}$$



For some $\sigma_l \in \mathbb{C}$ and $W^TV = I_r$

$$\begin{array}{rcl} \mathbf{span}\,(V) & \subseteq & \left[(\sigma_1 E - A)^{-1}\mathbf{b}, \dots, (\sigma_r E - A)^{-1}\mathbf{b}\right] \\ \mathbf{span}\,(W) & \subseteq & \left[(\sigma_1 E - A)^{-T}\mathbf{c}^T, \dots, (\sigma_r E - A)^{-T}\mathbf{c}^T\right] \end{array}$$

S. Gugercin and A C. Antoulas and C A. Beattie, *"$\mathcal{H}_2$ Model Reduction for Large Scale Linear Dynamical Systems"*, SIAM Journal on Matrix Analysis and Applications, 2008, Vol. 30(2), pp. 609-638.

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$$\mathbf{H}(s) = \sum_{j=1}^{n} \frac{\phi_j}{s - \lambda_j}$$

[a] $l = 1, \ldots, r$

If a realisation of $\mathbf{H}$ exists,

$$\mathbf{H}(s) = \mathbf{c}\,(sE - A)^{-1}\,\mathbf{b}$$

⇓ **Petrov-Galerkin (interp.)**

$$\hat{\mathbf{H}}(s) = \mathbf{c}V\left(sW^TEV - W^TAV\right)^{-1} W^T\mathbf{b}$$

⇓ **IRKA or TF-IRKA**

$\hat{\mathbf{H}}$ solves the $\mathcal{H}_2$ model approx. pb.

For $\sigma_l \leftarrow -\hat{\lambda}_l$ and $W^TV = I_r$

$$\begin{array}{rcl} \mathbf{span}\,(V) & \subseteq & \left[(-\hat{\lambda}_1 E - A)^{-1}\mathbf{b}, \ldots, (-\hat{\lambda}_r E - A)^{-1}\mathbf{b}\right] \\ \mathbf{span}\,(W) & \subseteq & \left[(-\hat{\lambda}_1 E - A)^{-T}\mathbf{c}^T, \ldots, (-\hat{\lambda}_r E - A)^{-T}\mathbf{c}^T\right] \end{array}$$

S. Gugercin and A C. Antoulas and C A. Beattie, *"$\mathcal{H}_2$ Model Reduction for Large Scale Linear Dynamical Systems"*, SIAM Journal on Matrix Analysis and Applications, 2008, Vol. 30(2), pp. 609-638.

Aircraft reduced order modelling

Focus on model approximation and reduction by interpolation (rational interpolation)

Given model $\mathbf{H}$, seek $\hat{\mathbf{H}}$ s.t.

$$\begin{array}{rcl}\hat{\mathbf{H}}(\mu_j) & = & \mathbf{H}(\mu_j)\\ \hat{\mathbf{H}}(\lambda_i) & = & \mathbf{H}(\lambda_i)\end{array}$$

$i = 1, \ldots, k$; $j = 1, \ldots, q$.

A.J. Mayo and A.C. Antoulas, *"A framework for the solution of the generalized realization problem"*, Linear Algebra and its Applications, 425(2-3), 2007, pp 634-662.

Aircraft reduced order modelling

Focus on model approximation and reduction by interpolation (rational interpolation)

Given model $\mathbf{H}$, seek $\hat{\mathbf{H}}$ s.t.

$$\begin{array}{rcl}\hat{\mathbf{H}}(\mu_j) & = & \mathbf{H}(\mu_j) \\ \hat{\mathbf{H}}(\lambda_i) & = & \mathbf{H}(\lambda_i)\end{array}$$

$i = 1, \ldots, k;\ j = 1, \ldots, q.$

$$\mathbb{L} = \begin{bmatrix} \frac{\mathbf{H}(\mu_1)-\mathbf{H}(\lambda_1)}{\mu_1-\lambda_1} & \cdots & \frac{\mathbf{H}(\mu_1)-\mathbf{H}(\lambda_k)}{\mu_1-\lambda_k} \\ \vdots & \ddots & \vdots \\ \frac{\mathbf{H}(\mu_q)-\mathbf{H}(\lambda_1)}{\mu_q-\lambda_1} & \cdots & \frac{\mathbf{H}(\mu_q)-\mathbf{H}(\lambda_k)}{\mu_q-\lambda_k} \end{bmatrix}$$

$$\mathbb{L}_\sigma = \begin{bmatrix} \frac{\mu_1\mathbf{H}(\mu_1)-\mathbf{H}(\lambda_1)\lambda_1}{\mu_1-\lambda_1} & \cdots & \frac{\mu_1\mathbf{H}(\mu_1)-\mathbf{H}(\lambda_k)\lambda_k}{\mu_1-\lambda_k} \\ \vdots & \ddots & \vdots \\ \frac{\mu_q\mathbf{H}(\mu_q)-\mathbf{H}(\lambda_1)\lambda_1}{\mu_q-\lambda_1} & \cdots & \frac{\mu_q\mathbf{H}(\mu_q)-\mathbf{H}(\lambda_k)\lambda_k}{\mu_q-\lambda_k} \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} \mathbf{H}(\lambda_1) & \ldots & \mathbf{H}(\lambda_k) \end{bmatrix}$$

$$\mathbf{V}^T = \begin{bmatrix} \mathbf{H}(\mu_1) & \ldots & \mathbf{H}(\mu_q) \end{bmatrix}$$

$\hat{\mathbf{H}}(s) = \mathbf{W}(-s\mathbb{L} + \mathbb{L}_\sigma)^{-1}\mathbf{V}$ $\Rightarrow$ Rational interpolation

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A.J. Mayo and A.C. Antoulas, *"A framework for the solution of the generalized realization problem"*, Linear Algebra and its Applications, 425(2-3), 2007, pp 634-662.

Aircraft reduced order modelling

Focus on model approximation and reduction by interpolation (Hermite interpolation)

Given model $\mathbf{H}$, seek $\hat{\mathbf{H}}$ s.t.

$$\begin{array}{rcl} \hat{\mathbf{H}}(\sigma_i) & = & \mathbf{H}(\sigma_i) \\ \hat{\mathbf{H}}'(\sigma_i) & = & \mathbf{H}'(\sigma_i) \end{array}$$

$i = 1, \dots, r.$

$$\mathbb{L} = \begin{bmatrix} \mathbf{H}'(\sigma_1) & \dots & \frac{\mathbf{H}(\sigma_1)-\mathbf{H}(\sigma_r)}{\sigma_1-\sigma_r} \\ \vdots & \ddots & \vdots \\ \frac{\mathbf{H}(\sigma_r)-\mathbf{H}(\sigma_1)}{\sigma_r-\sigma_1} & \dots & \mathbf{H}'(\sigma_r) \end{bmatrix}$$

$$\mathbb{L}_\sigma = \begin{bmatrix} (s\mathbf{H}(s))'_{s=\sigma_1} & \dots & \frac{\sigma_1\mathbf{H}(\sigma_1)-\sigma_r\mathbf{H}(\sigma_r)}{\sigma_1-\sigma_r} \\ \vdots & \ddots & \vdots \\ \frac{\sigma_r\mathbf{H}(\sigma_r)-\sigma_1\mathbf{H}(\sigma_1)}{\sigma_r-\sigma_1} & \dots & (s\mathbf{H}(s))'_{s=\sigma_r} \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} \mathbf{H}(\sigma_1) & \dots & \mathbf{H}(\sigma_r) \end{bmatrix}$$

$$\mathbf{V}^T = \begin{bmatrix} \mathbf{H}(\sigma_1) & \dots & \mathbf{H}(\sigma_r) \end{bmatrix}$$

$\hat{\mathbf{H}}(s) = \mathbf{W}(-s\mathbb{L} + \mathbb{L}_\sigma)^{-1}\mathbf{V}$ $\Rightarrow$ Hermite interpolation

A.J. Mayo and A.C. Antoulas, *"A framework for the solution of the generalized realization problem"*, Linear Algebra and its Applications, 425(2-3), 2007, pp 634-662.

Aircraft reduced order modelling

Back to the gust load models #1

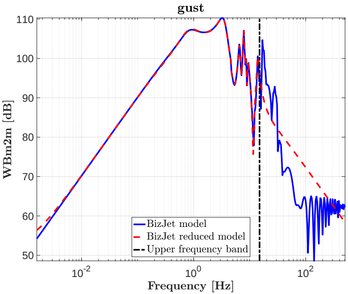
Large aeroelastic models

- $\approx$ 300 state variables
- Irrational and polynomial
- Not suitable for analysis and control

$\Longrightarrow$

Small and accurate models

- $\approx$ 30 state variables
- Rational (red over frequency range)
- Suitable for analysis and control



$$\mathbf{H}(s) = C\left(sE - A_0 - A_1 e^{\tau_1 s} - A_2 e^{\tau_2 s}\right)^{-1} B$$

$$\mathbf{H}(s) = \mathbf{H}_{\text{ODE irrat.}} + \mathbf{P}_{\text{Poly.}}$$

$\Downarrow$ (Rational approximation)

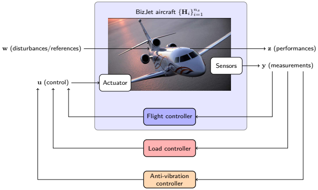
$$\mathbf{H}(s) = C(sE - A)^{-1} B.$$

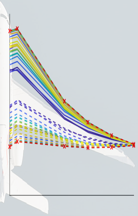
$\Downarrow$ (Reduction)

$$\hat{\mathbf{H}}(s) = \hat{C}(s\hat{E} - \hat{A})^{-1} \hat{B}.$$

Dynamical model → **Optimisation, control, ...** → A/C implementation and tests → Tests exploitation

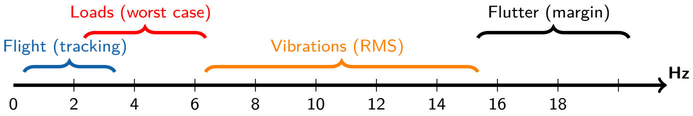
1. Aircraft gust load and vibration control problems
2. Aircraft large-scale modelling, model approximation and reduction
3. **Aircraft gust load and vibration control design in a industrial setup**
4. Aircraft control implementation and uncertainty analysis
5. Aircraft ground and flight experimental validations and data-driven modelling





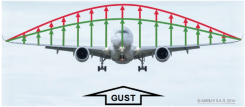
Aircraft feedback control design

Control objectives



#1 Gust load objective

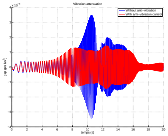
- Reduce the load envelope in response to a time-domain gust disturbance family
- Do not modify the flight controller performances



[Airbus image]

#2 Vibration attenuation objective

- Reduce the RMS vibrations around
- Do not modify the flight and load controllers performances

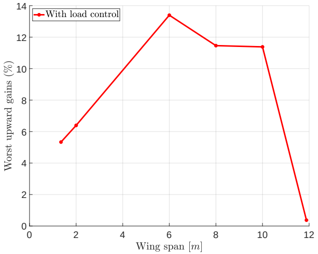


Aircraft feedback control design

Some results

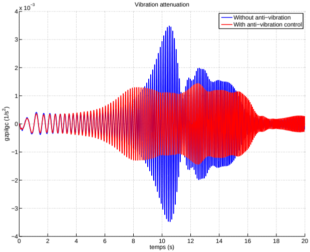
#1 Gust load control

From 5% to 14% of load attenuation



#2 Vibration control

Around 45% of vibration reduction



Notable reductions in loads and vibrations

C. P-V., P. Vuillemin, O. Cantinaud and F. Sève, *"Interpolatory Methods for Business Jet Gust Load Alleviation Function: Modelling, Control Design and Numerical Validation"*, under submission.

Aircraft feedback control design

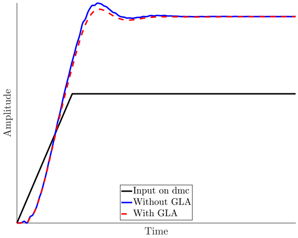
Some results

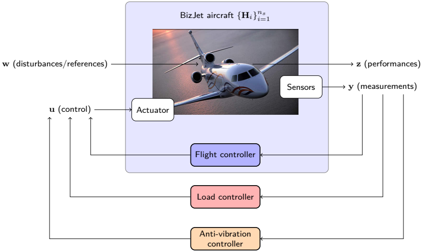
#1 Gust load control

From 5% to 14% of load attenuation

#2 Vibration control

Around 45% of vibration reduction





... are obtained without affecting the Flight control performances

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C. P-V., P. Vuillemin, O. Cantinaud and F. Sève, *"Interpolatory Methods for Business Jet Gust Load Alleviation Function: Modelling, Control Design and Numerical Validation"*, under submission.

Aircraft feedback control design

Glimpse of the considered control design approach (step 1)

The control problem paradigm

$$\mathbf{K}^\star(s) := \arg\min_{\mathbf{K}\in\mathcal{K}} \left|\left|\mathcal{F}_l\left(\mathbf{T}(s), \mathbf{K}(s)\right)\right|\right|_{\mathcal{H}_\infty}$$

- $\mathbf{T}(s) = \mathbf{W}_i(s)\hat{\mathbf{H}}(s)\mathbf{W}_o(s)$ is the generalised plant
- $\mathbf{K} \in \mathcal{K} \subseteq \mathcal{H}_\infty$ is the controller dynamical operator to be found.

$$\mathbf{K}(s) := \mathcal{F}_u\left(K, \frac{1}{s}I_{n_K}\right),$$

$$K = \left[\begin{array}{cc} A_K & B_u \\ C_y & D_{yu} \end{array}\right]$$

... is any matrix

Provide the "best" achievable performances

P. Apkarian and D. Noll, *"Nonsmooth $\mathcal{H}_\infty$ synthesis"*, IEEE Transactions on Automatic Control, 2006, Vol. 51(1), pp. 71-86..

Aircraft feedback control design

Glimpse of the considered control design approach (step 2)

Handle controllers "intersection" (given $\mathbf{K}^\star(s)$)

$$\mathbf{K}_\star(s) := \arg\min_{\mathbf{K}\in\mathcal{K}} \left|\left|\mathcal{F}_l\Big(\mathbf{T}(s),\mathbf{K}^\star(s)\Big) - \mathcal{F}_l\Big(\mathbf{T}(s),\mathbf{K}(s)\Big)\right|\right|_{\mathcal{H}_\infty}$$

- $\mathcal{F}_l\Big(\mathbf{T}(s),\mathbf{K}^\star(s)\Big)$ is the closed-loop obtained in step 1
- $\mathbf{K}\in\mathcal{K}\subseteq\mathcal{H}_\infty$ is the controller dynamical operator to be found.

$$\mathbf{K}(s) := \mathcal{F}_u\left(K, \frac{1}{s}I_{n_K}\right),$$

$$K = \left[\begin{array}{cc} A_K & B_u \\ C_y & D_{yu} \end{array}\right]$$

... is NOT any matrix...

$$A_K = \begin{pmatrix} \times & \times & \ldots & & \ldots & 0 \\ \times & \times & 0 & & \ldots & 0 \\ 0 & \times & \times & 0 & \ldots & 0 \\ \vdots & & \ddots & \ddots & \ddots & \\ \vdots & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & & \times & 0 \\ 0 & \ldots & 0 & \times & \times & \times \\ 0 & \ldots & & 0 & \times & \times \end{pmatrix}$$

P. Apkarian and D. Noll, *"Nonsmooth $\mathcal{H}_\infty$ synthesis"*, IEEE Transactions on Automatic Control, 2006, Vol. 51(1), pp. 71-86..

Aircraft feedback control design

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$$\mathbf{K}_{\star}(s) := \arg \min_{\mathbf{K} \in \mathcal{K}} \left|\left| \mathcal{F}_l\Big(\mathbf{T}(s), \mathbf{K}^{\star}(s)\Big) - \mathcal{F}_l\Big(\mathbf{T}(s), \mathbf{K}(s)\Big) \right|\right|_{\mathcal{H}_\infty}$$

- $\mathcal{F}_l\Big(\mathbf{T}(s), \mathbf{K}^{\star}(s)\Big)$ is the closed-loop obtained in step 1
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$$\mathbf{K}(s) := \mathcal{F}_u\left(K, \frac{1}{s} I_{n_K}\right),$$

$$K = \begin{bmatrix} A_K & B_u \\ C_y & D_{yu} \end{bmatrix}$$

... is NOT any matrix...

$$A_K =$$

$$\begin{pmatrix} \times & \times & \dots & & \dots & 0 \\ \times & \times & 0 & & \dots & 0 \\ 0 & \times & \times & 0 & \dots & 0 \\ \vdots & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & & \times & 0 \\ 0 & \dots & 0 & \times & \times & \times \\ 0 & \dots & & 0 & \times & \times \end{pmatrix}$$

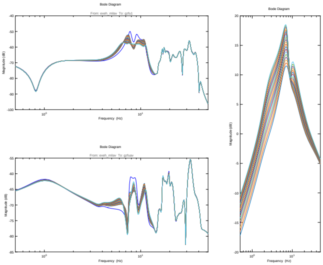
P. Apkarian and D. Noll, *"Nonsmooth $\mathcal{H}_\infty$ synthesis"*, IEEE Transactions on Automatic Control, 2006, Vol. 51(1), pp. 71-86..

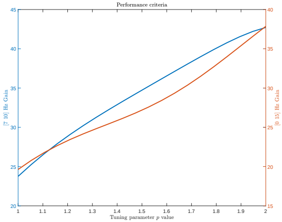
Aircraft feedback control design

Glimpse of the considered control design approach (go to parametric, for the vibration case #2)

The approach can be extended to parametric

Find $\mathbf{K}_\star(s, \mathbf{p})$, a parametric law, that can be adjusted on-line.





C. P-V., C. Leclercq and D. Sipp, *"Structured linear fractional parametric controller $\mathcal{H}_\infty$ design and its applications"*, in Proceedings of the European Control Conference (ECC), Limassol, Cyprus, June, 2018, pp. 2629-2634.

Aircraft feedback control design

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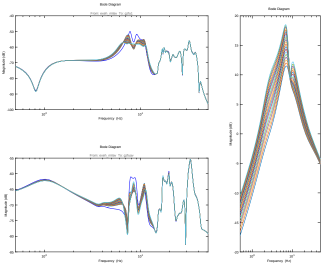
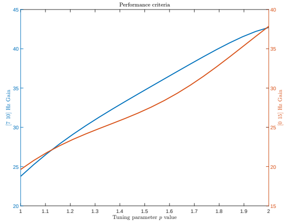
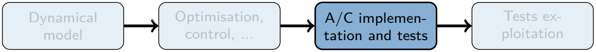




As a function of p, the vibration attenuation is better and better, but requires more actuator activity (used for testing)

C. P-V., C. Leclercq and D. Sipp, *"Structured linear fractional parametric controller $\mathcal{H}_\infty$ design and its applications"*, in Proceedings of the European Control Conference (ECC), Limassol, Cyprus, June, 2018, pp. 2629-2634.



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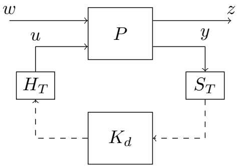
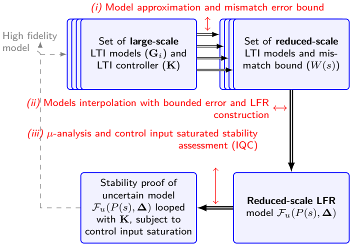
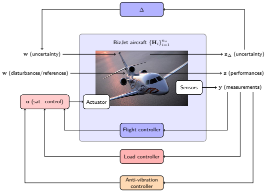


Aircraft implementation and analysis

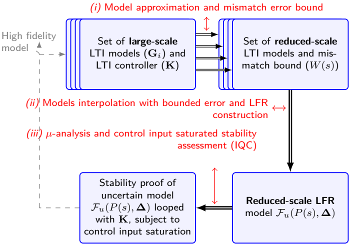
Uncertainty and actuator saturation analysis

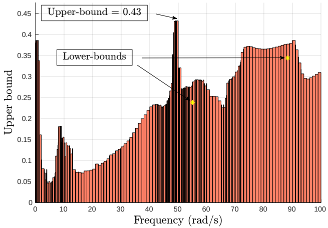


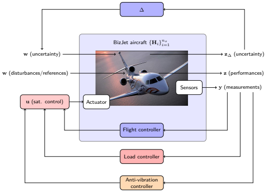


Aircraft implementation and analysis

Uncertainty and actuator saturation analysis



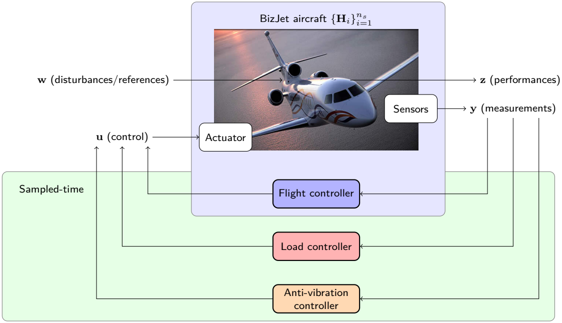




https://w3.onera.fr/smac/

Aircraft implementation and analysis

Digital control design



$\mathbf{K}(s) \rightarrow \mathbf{K}_d(z)$ and implementation constrains

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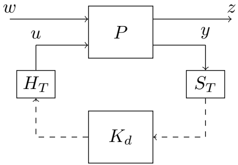
P. Vuillemin and C. P-V., *"Discretisation of continuous-time linear dynamical model with the Loewner interpolation framework"*, initial version available as arXiv:1907.10956.

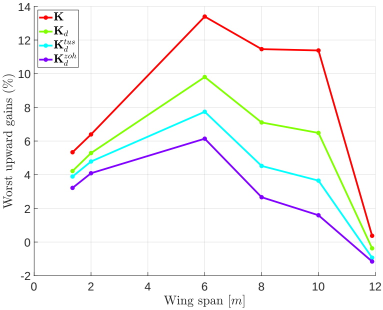
C. P-V., P. Vuillemin, F. Sève and O. Cantinaud, *"Interpolatory Methods for Business Jet Gust Load Alleviation Function: Modelling, Control Design and Numerical Validation"*, under submission.

Aircraft implementation and analysis

Control laws are discretised

LTI controller is discretised at fixed period with **Tustin**, **ZOH** and **Loewner $\mathbf{K}_d$**.





P. Vuillemin and C. P-V., *"Discretisation of continuous-time linear dynamical model with the Loewner interpolation framework"*, initial version available as arXiv:1907.10956.

Dynamical model → Optimisation, control, ... → A/C implementation and tests → **Tests exploitation**

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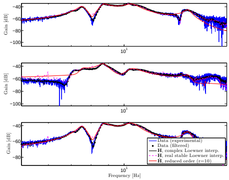
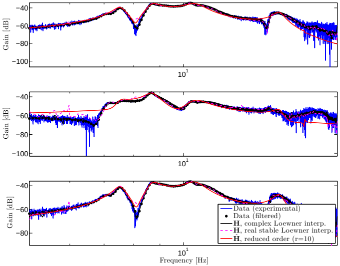
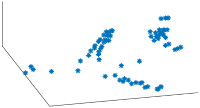


Aircraft experimental validation

Ground test validation by Dassault-Aviation, toward data-driven interpolation



- Ground test at Dassault-Aviation
- Measurements at #100 points
- Application of **data-driven interpolatory methods**



C. Meyer, J. Prodigue, G. Broux, O. Cantinaud and C. P-V., *"Ground test for vibration control demonstrator"*, in Proceedings of the MOVIC & RASD, Southampton, UK, 2016.

Aircraft experimental validation

Model approximation/reduction by interpolation (rational interpolation reminder)

Given model $\mathbf{H}$, seek $\hat{\mathbf{H}}$ s.t.

$$\begin{array}{rcl}\hat{\mathbf{H}}(\mu_j) & = & \mathbf{H}(\mu_j)\\ \hat{\mathbf{H}}(\lambda_i) & = & \mathbf{H}(\lambda_i)\end{array}$$

$i = 1, \ldots, k;\ j = 1, \ldots, q.$

$$\mathbb{L} = \begin{bmatrix} \frac{\mathbf{H}(\mu_1)-\mathbf{H}(\lambda_1)}{\mu_1-\lambda_1} & \cdots & \frac{\mathbf{H}(\mu_1)-\mathbf{H}(\lambda_k)}{\mu_1-\lambda_k} \\ \vdots & \ddots & \vdots \\ \frac{\mathbf{H}(\mu_q)-\mathbf{H}(\lambda_1)}{\mu_q-\lambda_1} & \cdots & \frac{\mathbf{H}(\mu_q)-\mathbf{H}(\lambda_k)}{\mu_q-\lambda_k} \end{bmatrix}$$

$$\mathbb{L}_\sigma = \begin{bmatrix} \frac{\mu_1\mathbf{H}(\mu_1)-\mathbf{H}(\lambda_1)\lambda_1}{\mu_1-\lambda_1} & \cdots & \frac{\mu_1\mathbf{H}(\mu_1)-\mathbf{H}(\lambda_k)\lambda_k}{\mu_1-\lambda_k} \\ \vdots & \ddots & \vdots \\ \frac{\mu_q\mathbf{H}(\mu_q)-\mathbf{H}(\lambda_1)\lambda_1}{\mu_q-\lambda_1} & \cdots & \frac{\mu_q\mathbf{H}(\mu_q)-\mathbf{H}(\lambda_k)\lambda_k}{\mu_q-\lambda_k} \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} \mathbf{H}(\lambda_1) & \ldots & \mathbf{H}(\lambda_k) \end{bmatrix}$$

$$\mathbf{V}^T = \begin{bmatrix} \mathbf{H}(\mu_1) & \ldots & \mathbf{H}(\mu_q) \end{bmatrix}$$

$\hat{\mathbf{H}}(s) = \mathbf{W}(-s\mathbb{L} + \mathbb{L}_\sigma)^{-1}\mathbf{V}$ $\Rightarrow$ Rational interpolation

A.J. Mayo and A.C. Antoulas, *"A framework for the solution of the generalized realization problem"*, Linear Algebra and its Applications, 425(2-3), 2007, pp 634-662.

Aircraft experimental validation

Model approximation/reduction by interpolation (data-driven interpolation)

Given $\{\mu_j, \mathbf{v}_j\}$, $\{\lambda_i, \mathbf{w}_i\}$ data, seek $\hat{\mathbf{H}}$, s.t.

$$
\begin{array}{rcl}
\hat{\mathbf{H}}(\mu_j) & = & \mathbf{v}_j \\
\hat{\mathbf{H}}(\lambda_i) & = & \mathbf{w}_i
\end{array}
$$

$i = 1, \dots, k$; $j = 1, \dots, q$.

$$
\mathbb{L} = \begin{bmatrix}
\frac{\mathbf{H}(\mu_1)-\mathbf{H}(\lambda_1)}{\mu_1-\lambda_1} & \dots & \frac{\mathbf{H}(\mu_1)-\mathbf{H}(\lambda_k)}{\mu_1-\lambda_k} \\
\vdots & \ddots & \vdots \\
\frac{\mathbf{H}(\mu_q)-\mathbf{H}(\lambda_1)}{\mu_q-\lambda_1} & \dots & \frac{\mathbf{H}(\mu_q)-\mathbf{H}(\lambda_k)}{\mu_q-\lambda_k}
\end{bmatrix}
$$

$$
\mathbb{L}_\sigma = \begin{bmatrix}
\frac{\mu_1\mathbf{H}(\mu_1)-\mathbf{H}(\lambda_1)\lambda_1}{\mu_1-\lambda_1} & \dots & \frac{\mu_1\mathbf{H}(\mu_1)-\mathbf{H}(\lambda_k)\lambda_k}{\mu_1-\lambda_k} \\
\vdots & \ddots & \vdots \\
\frac{\mu_q\mathbf{H}(\mu_q)-\mathbf{H}(\lambda_1)\lambda_1}{\mu_q-\lambda_1} & \dots & \frac{\mu_q\mathbf{H}(\mu_q)-\mathbf{H}(\lambda_k)\lambda_k}{\mu_q-\lambda_k}
\end{bmatrix}
$$

$$
\mathbf{W} = \begin{bmatrix} \mathbf{H}(\lambda_1) & \dots & \mathbf{H}(\lambda_k) \end{bmatrix}
$$

$$
\mathbf{V}^T = \begin{bmatrix} \mathbf{H}(\mu_1) & \dots & \mathbf{H}(\mu_q) \end{bmatrix}
$$

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$i = 1, \dots, k$; $j = 1, \dots, q$.

$$\mathbb{L} = \begin{bmatrix} \frac{\mathbf{v}_1 - \mathbf{w}_1}{\mu_1 - \lambda_1} & \dots & \frac{\mathbf{v}_1 - \mathbf{w}_k}{\mu_1 - \lambda_k} \\ \vdots & \ddots & \vdots \\ \frac{\mathbf{v}_q - \mathbf{w}_1}{\mu_q - \lambda_1} & \dots & \frac{\mathbf{v}_q - \mathbf{w}_k}{\mu_q - \lambda_k} \end{bmatrix}$$

$$\mathbb{L}_\sigma = \begin{bmatrix} \frac{\mu_1 \mathbf{v}_1 - \mathbf{w}_1 \lambda_1}{\mu_1 - \lambda_1} & \dots & \frac{\mu_1 \mathbf{v}_1 - \mathbf{w}_k \lambda_k}{\mu_1 - \lambda_k} \\ \vdots & \ddots & \vdots \\ \frac{\mu_q \mathbf{v}_q - \mathbf{w}_1 \lambda_1}{\mu_q - \lambda_1} & \dots & \frac{\mu_q \mathbf{v}_q - \mathbf{w}_k \lambda_k}{\mu_q - \lambda_k} \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} \mathbf{w}_1 & \dots & \mathbf{w}_k \end{bmatrix}$$

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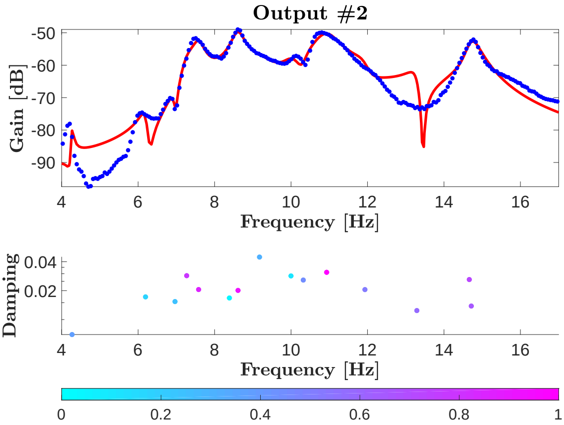
$\hat{\mathbf{H}}(s) = \mathbf{W}(-s\mathbb{L} + \mathbb{L}_\sigma)^{-1}\mathbf{V}$ $\Rightarrow$ Rational interpolation (data driven)

A.J. Mayo and A.C. Antoulas, *"A framework for the solution of the generalized realization problem"*, Linear Algebra and its Applications, 425(2-3), 2007, pp 634-662.

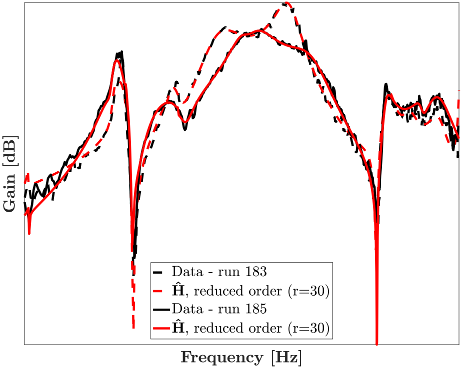
Aircraft experimental validation

Ground vibration test iterpolatory results



Aircraft experimental validation

Ground vibration test iterpolatory results with control



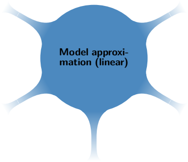
Data vs. **Model** & open-loop (dashed) vs. closed-loop (solid)

Vibration attenuation validated in GVT... then in flight test $\Rightarrow$ Video

Conclusions

What to keep in mind...

Importance of dynamical data science within industry
Illustration on a Dassault Aviation Business Jet aircraft



C. P-V., *"Large-scale dynamical model approximation and its applications"*, HDR thesis, INP Toulouse, Université de Toulouse, ONERA, July 2019 (on-line available).

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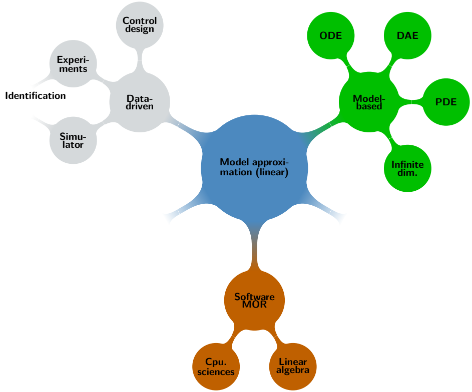

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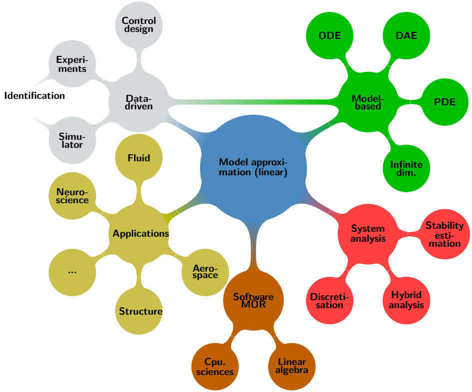

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Conclusions

... a versatile tool

Interpolation-based methods are remarkably versatile and indicated for

- model reduction and approximation,
- but also complex function analysis ...

$\rightarrow$ **direct impact in aircraft engineers life, but not only**

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- **MOR Toolbox integrated tool at** `http://mordigitalsystems.fr/`
- **More references at** `https://sites.google.com/site/charlespoussotvassal/`

mor Digital Systems

"Merge data and physics using computational sciences and engineering, for enhanced dynamical systems experience"

Conclusions

Acknowledgements and on-going projects

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- O. Kocan [learning control]

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- Test team [ground/flight tests]

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Civil aircraft reduced order modelling, control and validation: Numerical and experimental challenges

... how much dynamical data science is a strong ally?

Charles Poussot-Vassal
Friday September 18, 2020
28th IEEE Mediterranean Conference on Control and Automation Conference
(e-) St Raphael, France
FrPL3 Plenary Session, ROOM SR1

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